NONLINEAR THEORY OF THE PROPAGATION OF PULSES OF ELECTROMAGNETIC WAVES THROUGH A PLASMA BOUNDARY

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A solution is obtained for the problem of the propagation of electromagnetic waves of arbitrary form through a plasma boundary on condition that the length of the wave train is much greater than the wave length. A solution is found both for the case of a wide spectrum of width $\Delta\omega$ much greater than the plasma frequency ω_0, as well as for a narrow spectrum. The results obtained enable us to draw conclusions about the time and space variation of the shape of electromagnetic pulses in a plasma.

The passage of high frequency electromagnetic waves through a plasma is similar to that of a beam of charged particles [1, 2]. This is associated with the fact that decay processes are similar to Cerenkov radiation effects. The dynamics of the development of transverse wave instabilities in a uniform isotropic plasma were studied in [2] assuming that the wave phase behaves stochastically. It was calculated here that instabilities develop quite differently in the case of a wide frequency spectrum than in the case of a narrow "monochromatic" spectrum. If we can speak of transverse quanta diffusion effects in the "field" of the generated longitudinal quanta in the first case, and if the resulting effects are closely similar to the nonlinear effects arising when beam instability develops [3, 4], then the development of instabilities in the case of a narrow spectrum leads to the appearance of red satellites in the transverse wave spectrum differing from the basic frequency ω by a quantity $\nu\omega_0$ ($\nu = 1, 2, 3, \ldots$). In this case the development of the instability corresponds to a tendency for a plateau over the satellites to appear.

Attention should however be drawn to the fact that the dynamics of instability development in a semibounded plasma may be quite different. This is associated first with the different values of group velocities of transverse and longitudinal waves, and what is also important, with the effect of longitudinal wave accumulation in the boundary region if the length of the wave train is sufficiently large. The treatment of a similar problem for beam instabilities in paper [5] showed that a narrow transition layer may arise with a transverse wave energy density greatly in excess of the energy density of the injected beam. In what follows we examine the part played by boundary effects in the passage of pulses of electromagnetic waves through the boundary of the plasma. The cases of both narrow and wide spectra are considered. We note that in the case of narrow spectra the wave train must necessarily be greatly in excess of $\Delta\omega^{-1}$, and the effects of the accumulation of oscillations will be appreciable.

The phases of both transverse waves, and also generated longitudinal waves are assumed to be stochastic quantities. The boundary effects which have been treated may be applied both in the generation of longitudinal waves necessary for the effective acceleration of particles in a plasma as well as in the modulation and alteration of the initial transverse wave spectrum. It should also be stressed that these effects which have been considered could be applied for turbulent plasma diagnostics, as has already been pointed out in [2].

§1. The dynamics of the passage of "monochromatic" electromagnetic wave pulses through a plasma boundary. 1°. We shall consider the interaction dynamics of the spectrum of a transverse wave impulse under the conditions $\Delta\omega \ll \omega_0$, confining ourselves to effects associated with the appearance of one of the satellites $\omega \pm \omega_0$. Strictly speaking, such an approximation is valid only in a one dimensional model, since in this case the longitudinal waves generated on the appearance of the satellite $\omega \pm \omega_0$, do not participate in a further transfer of energy through the spectrum. A similar situation is possible only when there is a strong magnetic field present in the plasma, in the direction of the beam of incident transverse waves [7].

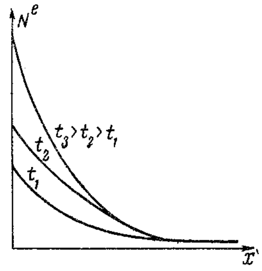

Fig. 1. The variation with time of the spatial distribution of longitudinal noise in a plasma in the case when the incident wave is N_0.

Under the conditions $\Delta\omega \ll \omega_0$ the equations of paper [2] describing the dynamics for the appearance of the first satellite may be written in the form

$$\frac{\partial N_0}{\partial t} + V_0 \frac{\partial N_0}{\partial x} = \alpha N^l (N_0 - N_1) \qquad \left(\alpha = \frac{e^2\omega_0^3}{8\pi m_e^2 k_l^2}\right), \quad (1.1)$$

$$\frac{\partial N_1}{\partial t} + V_1 \frac{\partial N_1}{\partial x} = \alpha N^l (N_1 - N_0), \quad (1.2)$$

$$\frac{\partial N^l}{\partial t} = \beta N^l (N_0 - N_1) \qquad \left(\beta = \alpha \frac{k_l^3}{\omega_0^3}\right) \quad (1.3)$$

when the variation of wave intensity in space is taken into account (the velocity of light is taken to be $c = 1$).

Here $N_0(t, x, k_t)$, $N_1(t, x, k_t)$ are one-dimensional wave distribution functions for the fundamental frequency and the first satellite $\omega - \omega_0$; V_0 and V_1 are the group velocities of the waves ω and $\omega - \omega_0$, N^l is a one-dimensional distribution function for longitudinal waves, k_t here and in what follows is the transverse wave number, while k_l is the longitudinal wave number. Compared with Eqs. (49)–(51) of paper [2], the terms N_0N_1 are not taken into account. This turns out to be valid for $\omega \gg \omega_0$ with an accuracy to the order of terms $(\omega_0/\omega)^3$. Moreover terms with $V^l \partial N^l/\partial x$ are omitted here where V^l is the group velocity of longitudinal waves since it is assumed that the electromagnetic wave pulse is sufficiently short so that in the time for the waves to pass through the plasma the longitudinal wave energy cannot be transferred over a notable distance from the region where they were excited by transverse waves. Moreover in view of the inequality $\omega \gg \omega_0$ we have $V_1 \approx V_0 \approx V \approx 1$, i.e., the group velocities of the high-frequency transverse waves differ little from the velocity of light.

In this case it follows from (1.1), (1.2) that

$$\left(\frac{\partial}{\partial t} + V\frac{\partial}{\partial x}\right)(N_0 + N_1) = 0, \qquad (1.4)$$

which gives

$$N_0(t, x) + N_1(t, x) = A(\xi), \qquad \xi = x - Vt. \quad (1.5)$$

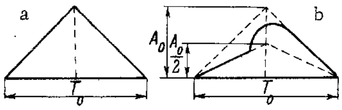

Fig. 2. The change of pulse shape N_0 on passing through the plasma: a) pulse shape on entering the plasma; b) pulse shape on leaving the plasma.

We shall assume that for t = 0 the leading edge of the pulse reaches the plasma boundary (x = 0). If we are interested in the effect occurring in a relatively quiet plasma at t = 0 in which the noise intensity is sufficiently small, i.e.,

$$N^l(0) \ll \frac{\beta}{\alpha} N^t(0), \qquad \text{or} \quad W^l(0) \ll \frac{\omega_0}{\omega} W^t(0), \quad (1.6)$$

then the development is accompanied by the transition $N_0 \to N_1$, and the wave N_0 must be taken as the incident wave, i.e.,

$$A(x) = N_0(0, x). \qquad (1.7)$$

However, if we are interested in the effect arising in an excited turbulent plasma when the inverse inequality to (1.6) is fulfilled, then the development of instabilities corresponds to the transitions $N_1 \to N_0$, and

$$A(x) = N_1(0, x). \qquad (1.8)$$

The formal solution of Eq. (1.3) has the form

$$N^l(t, x) = N^l(0, x)\exp\left[\beta\int_0^t (N_0 - N_1)\,dt'\right].$$

We shall assume that the initial noise is homogeneous for x > 0; then

$$N^l(0, x) = \theta(x) N_i^l, \quad N_i^l = \text{const}, \qquad \theta(x) = \begin{cases} 1 & (x > 0), \\ 0 & (x < 0). \end{cases} \quad (1.9)$$

We shall investigate the solution for x > 0. We introduce

$$z(t, x) = \int_0^t (N_0(t', x) - N_1(t', x))\,dt'. \qquad (1.10)$$

It follows from (1.5), (1.10) that

$$N_0(t, x) = \frac{A(\xi)}{2} + \frac{\partial z}{\partial t}. \qquad (1.11)$$

Setting (1.11) in (1.1) and allowing for the fact that $(\partial/\partial t + V\partial/\partial x)A(\xi) = 0$, we obtain

$$\frac{\partial}{\partial t}\left\{\frac{\partial z}{\partial t} + V\frac{\partial z}{\partial x} + \frac{\alpha}{\beta} N_i^l \exp(\beta z)\right\} = 0. \qquad (1.12)$$

We may easily find the constant in the conservation law if we remember that z = 0, $\partial z/\partial t = 0$ for x > 0, t = 0:

$$\frac{\partial z}{\partial t} + V\frac{\partial z}{\partial x} + \frac{\alpha}{\beta} N_i^l[\exp(\beta z) - 1] = 0. \qquad (1.13)$$

The solution of (1.13) has the form

$$\ln[1 - \exp(-\beta z)] = -2\alpha N_i^l t + \Lambda(\xi). \qquad (1.14)$$

The function $\Lambda(\xi)$ must be determined from the boundary condition.

2°. Let a pulse N_0 be incident on a plasma with a low initial level of longitudinal noise. We then have

$$z(t, 0) = -\frac{1}{V}\int_0^{-Vt} A(\xi)\,d\xi \qquad (x = 0).$$

From here

$$\Lambda(\xi) = -\frac{2\alpha N_i^l}{V}\xi + \ln[1 - \psi(\xi)],$$

$$\psi(\xi) = \exp\frac{\beta}{V}J(\xi), \qquad J(\xi) = \int_0^{\xi} A(\xi')\,d\xi'. \quad (1.15)$$

The solution will have the form

$$N_0(t, x) = \frac{{}^1\!/_2 A(\xi)(1 - s(x)) + A(\xi)\psi(\xi)s(x)}{1 - s(x) + \psi(\xi)s(x)},$$

$$s(x) = \exp\left(-\frac{2\alpha}{V}N_i^l x\right). \qquad (1.16)$$

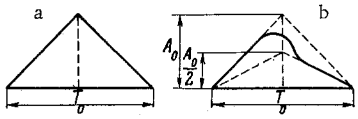

Fig. 3. The change of pulse shape N_1 on passing through the plasma: a) pulse shape on entering the plasma; b) pulse shape on leaving the plasma.

The result obtained (1.16) describes the change of shape of the pulse in space and time. It is also convenient to write the expression for the change in intensity of longitudinal plasma waves

$$N^l(t, x) = \frac{N_i^l}{1 - s(x) + \psi(\xi)s(x)}. \qquad (1.17)$$

For not very large x ($x \ll Vt$, $x \ll V/\alpha N_i^l$) this formula simplifies and assumes the form

$$N^l(t, x) = \frac{\beta}{2\alpha}A_0\frac{\Delta_0}{x + \Delta_0\exp[-\gamma(\tau - \tau^\circ)]}. \qquad (1.18)$$

Here

$$\tau = \frac{1}{A_0 V}\int_{-V_0 t}^{0} A(\xi')\,d\xi' = -\frac{1}{A_0\beta}\ln\psi(-Vt),$$

$$\gamma = \beta A_0, \qquad \tau^\circ = \frac{1}{\gamma}\ln\frac{\beta A_0}{2\alpha N_i^{\,l}}.$$

Here τ° is the relaxation time for the transverse wave pulse in an unbounded plasma, A_0 is the maximum value of A in the pulse. The energy distribution of plasma oscillations as a function of x and t is shown in Fig. 1 for $t > \tau^\circ$; N^l is maximum in a narrow layer Δ close to the plasma boundary

$$\Delta = \Delta_0 \exp\left[-\gamma(\tau - \tau^\circ)\right].$$

For $t > \tau^\circ$ this layer migrates according to an exponential law towards the left plasma boundary as time passes. The quantity N^l also increases exponentially with time in this layer. The formulas obtained for N^l are applicable for t which are not very large:

$$\exp\left[-\gamma(\tau - \tau^\circ)\right] \gg \frac{V^l}{V}.$$

For large t the transfer of plasma oscillation energy becomes appreciable and this leads to saturation for

$$N^l \approx A^*\frac{V}{V^l} \gg A^* \qquad \left(A^* = \frac{\beta A_0}{2\alpha}\right).$$

Here A* is the maximum value of N^l in the development of instabilities in an unbounded plasma. In the case under consideration the maximum energy of longitudinal oscillations is of the order*

$$W^l \approx W^t\frac{\omega_0}{\omega}\frac{V}{V^l}.$$

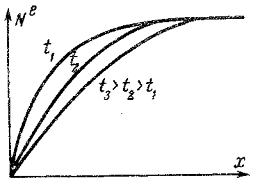

Fig. 4. The spatial distribution of longitudinal noise in a plasma for different values of t in the case when the incident wave is N_1.

*It should be noted that in some cases the saturation of longitudinal oscillation energy may be associated with nonlinear wave interaction, and in the first place with induced scattering of longitudinal oscillations of particles in the plasma leading to a transfer of energy from these oscillations to the nonresonance part of the spectrum. In these cases saturation will occur for smaller N^l.

If the plasma is bounded to a dimension of order L, then on leaving the plasma the pulse will be of the form

$$N_0^-(\xi) = A(\xi)\frac{{}^1\!/_2[1 - s(L)] + \psi(\xi)s(L)}{1 - s(L) + s(L)\psi(\xi)}. \qquad (1.19)$$

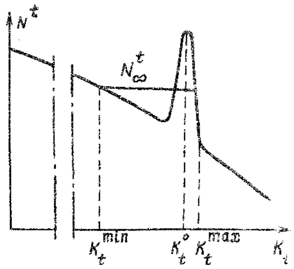

Fig. 5. The distribution function of transverse "noise," N^t_∞ is the stable distribution with a "plateau," $k_t^{\min}$, $k_t^{\max} = K_t^{\min} + \Delta k_t$ are the plateau boundaries.

We note that the variation of pulse shape is different in the two cases

$$\frac{2\alpha N_i^{\,l} L}{V} \gg 1, \qquad \frac{2\alpha N_i^{\,l} L}{V} \ll 1.$$

In the first case there is a proportional decrease in intensity with a negligible change of shape:

$$N_0^-(\xi) \approx \frac{1}{2}A(\xi) \quad \text{for} \quad \frac{2\alpha N_i^{\,l} L}{V} \gg 1.$$

In the second case a radical change of pulse shape is possible:

$$N_0^-(\xi) = A(\xi)\frac{\alpha N_i^{\,l} L/V + \psi(\xi)}{2\alpha N_i L/V + \psi(\xi)} \quad \text{for} \quad \frac{2\alpha N_i^{\,l}}{V}L \ll 1. \quad (1.20)$$

By way of an example we shall consider a triangular form pulse of length T_0, when $A(\xi)$ may be written in the form

$$A(\xi) = \begin{cases} 0 & (\xi < -VT_0), \\ 2A_0(\xi + VT_0)/VT_0 & (-VT_0 < \xi < -{}^1\!/_2 VT_0), \\ -2A_0\xi/VT_0 & (-{}^1\!/_2 VT_0 < \xi < 0), \\ 0 & (\xi > 0). \end{cases} \qquad (1.21)$$

Then the expression (1.15) for $J(\xi)$ is

$$-J(\xi) =$$

$$= \begin{cases} 0 & (\xi > 0), \\ A_0\xi^2/VT_0 & (-{}^1\!/_2 VT_0 < \xi < 0), \\ A_0V^{-1}T_0^{-1}\left[{}^1\!/_2 (VT_0)^2 - (\xi + VT_0)^2\right] & (-VT_0 < \xi < -{}^1\!/_2 VT_0), \\ {}^1\!/_2 A_0 VT_0 & (\xi < -VT_0). \end{cases} \qquad (1.22)$$

It follows from (1.20), (1.22) that the pulse shape remains quite unchanged in the region close to $\xi = 0$, then just as in the case when the pulse length is fairly large, the pulse decreases from A to (1/2)A (see Fig. 2) as ξ decreases.

Using (1.22) and (1.17) we obtain the following formula for the plasma wave distribution after the pulse has traversed the plasma:

$$N^l(x) = \frac{N_i^{\,l}}{1 - s(x) + s(x)\exp\left(-{}^1\!/_2\beta A_0 T_0\right)}. \qquad (1.23)$$

3°. We shall consider effects which arise when the incident wave is N_1. In this case, according to (1.7), (1.15), we obtain

$$z(t, 0) = \frac{1}{V} \int_0^{-Vt} A(\xi)\, d\xi,$$

$$N_1^-(\xi) = A(\xi) \frac{1/_2 [1 - s(L)] + s(L)\psi^{-1}(\xi)}{1 - s(L) + s(L)\psi^{-1}(\xi)}.$$

We can easily see that the picture which arises in this case is in some sense the opposite to that obtained above. Namely, for a low level of initial noise, i.e., for

$$\frac{2\alpha N_i^l}{V} L \ll 1 \qquad (1.24)$$

the changes of pulse intensity and shape are negligibly small. However by the inverse inequality to (1.24)

$$N_1^-(\xi) = A(\xi) \frac{1/_2 + s(L)\psi^{-1}(\xi)}{1 + s(L)\psi^{-1}(\xi)}. \qquad (1.25)$$

For $\xi = 0$ there is a maximum change in the pulse $N_1^-(\xi) \approx A(\xi)/2$, and for ξ which satisfy the relation

$$\frac{\beta}{V} \int_\xi^0 A(\xi')\, d\xi' \approx \frac{2\alpha N_i^l}{V} L \qquad (1.26)$$

the pulse shape approaches the initial (indicated in Fig. 3).

It should be stressed that in this case the characteristic ξ for which a change in the spectrum occurs depends on the initial energy of the plasma oscillations according to a power law, i.e., fairly strongly, while in the first case the dependence was logarithmic [compare (1.25) and (1.20)]. Finally for the variation in plasma wave intensity in the case under consideration we have, after the pulse has passed

$$N^l(x) = \frac{N_i^l}{1 - s(x) + s(x)\exp(1/_2\beta A_0 T_0)}. \qquad (1.27)$$

In the given case there is a decrease in the intensity of oscillations, while the maximum effect occurs for x = 0:

$$N^l(0) = N_i^l \exp(-1/_2\beta A_0 T_0).$$

If

$$1/_2\beta A_0 T_0 < 2\alpha N_i^l L / V, \qquad (1.28)$$

then there is a decrease to the value

$$x = \frac{\beta}{4\alpha} A_0 V T_0 (N_i^l)^{-1}.$$

If the inverse relation to (1.28) is fulfilled, the intensity of the oscillations decreases proportionally along the entire length of the plasma. It should be stressed that the longitudinal wave distribution within the plasma has a behavior in time which is the opposite to that considered in Paragraph 2 for the case of a low initial noise level of longitudinal oscillations. Namely (see Fig. 4), the region where the field of plasma oscillations has decreased considerably, is situated close to the boundary x = 0 and moves linearly

$$x = \frac{\beta A_0}{\beta A_0 + 2\alpha N_i^l} Vt$$

towards the other boundary of the plasma. This is explained by the fact that a beam of transverse waves on passing into the plasma leads to the damping of plasma oscillation energy for small x at first, and the remaining parts of the beam pulse pass through the regions of small x unhindered and lead to wave damping for large x. We note that linear movement occurs for those values of x for which $A(\xi) = \mathrm{const}(A = A_0)$.

§2. The passage of blurred pulses of electromagnetic waves through a plasma division boundary. 1°. In the case where the incident pulse has a wide frequency spectrum ($\Delta\omega \gg \omega_0$) the diffusion approximation must be used to describe the change in transverse $N^t(k_t)$ and longitudinal $N^l(k_l)$ wave numbers. The system of equations in this case is similar to the quasi-linear system

$$\frac{\partial N^t}{\partial t} + V\frac{\partial N^t}{\partial x} = \frac{e^2\omega_0^5}{8\pi m_e^2}\frac{\partial}{\partial k_t}\left(\frac{N^l}{k_l^2}\frac{\partial N^t}{\partial k_t}\right),$$
$$\frac{\partial N^l}{\partial t} = \frac{e^2\omega_0 k_t}{8\pi m_e^2} N^l \frac{\partial N^t}{\partial k_t}. \qquad (2.1)$$

As in the preceding paragraph, in (2.1) terms proportional to N^tN^t and $V^l\partial N^l/\partial x$ are neglected; k_t and k^l in (2.1) are connected by the law of conservation of energy on decay:

$$k_l V = \omega_0, \qquad \left(V = \frac{k_t}{\sqrt{k_t^2 + \omega_0^2}}\right). \qquad (2.2)$$

Equations (2.1) are similar to those investigated in paper [5]. We shall thus confine ourselves to a brief exposition of the results which may be obtained in a similar manner to those in paper [5]. Let the spectrum of transverse waves incident on the plasma have a maximum for $k_t \gg k_t^* \gg T/\hbar$ (Fig. 5).

The energy integral of system (2.1) is used, and keeping in mind that the incident pulse has a triangular form of length t_0

$$N^t(t, 0) = N_0^t [\theta(t_0 - t) - \theta(-t)] \qquad (2.3)$$

we obtain for sufficiently large t ($t > \tau_0$) the following relation for the energy of the oscillations concentrated in the layer $0 < x < \Delta_0$:

$$\int_0^{\Delta_0} N^l dx = \frac{k_t^3}{\omega_0^4} \int_{k_t^{\min}}^{k_t} (N_\infty^t - N_0^t)\, dk_t' [t_0 + (t - t_0)\theta(t_0 - t)],$$

$$\gamma = \frac{e^2\omega_0 k_t W_0^t}{8\pi m_e^2 \Delta k_t}, \qquad \Delta_0 = V\tau_0,$$

$$\tau_0 = \frac{1}{\gamma}\ln\frac{W_\infty}{W_0^l}, \qquad W_\infty^l = \frac{\Delta k_t}{k_t} W_0^t. \qquad (2.4)$$

Here τ_0 is the pulse relaxation time in an unbounded plasma, γ is the time increment, W_0^l and W_∞^l are the initial and final energies of the plasma waves for the development of an instability in an unbounded plasma, N^t_∞ is the stable transverse wave distribution with a plateau (Fig. 5).

Thus the mean energy of plasma waves for $t < t_0$ increases linearly with t and for $t > \tau_0$ is greater by a factor t/τ_0 than the quantity

$$N^l = \frac{k_t^3}{\omega_0^4} \int\limits_{k_t^{\min}}^{k_t} (N_\infty{}^t - N_0{}^t)\, dk_t'$$

corresponding to the level of longitudinal noise generated in an unbounded plasma. As the plasma oscillation energy increases the length Δ over which pulse relaxation occurs should decrease, while just as in [5] and in the case considered in §1, the variation of Δ with time is determined by the approximate formula

$$\Delta = \Delta_0 \exp\left[-\gamma(t - \tau_0)\right] \quad (t < t_0). \qquad (2.5)$$

It should be stressed that just as in the case of a discrete spectrum the distribution function N^t which decreases monotonically with k_t may become unstable in the presence of intense longitudinal noise. The development of instability leads to the appearance of a plateau in the region k_t in which there was intense longitudinal noise.

2°. In the case when the interval k_t in which a change in the transverse wave spectrum occurs, is sufficiently small ($\Delta k_t \ll k_t$), we may obtain a similar solution to the problem. We shall seek the solution of (2.1) in the form

$$N^t = N^t_\infty + B(t, x)(k_t - k_t^\circ),$$
$$N^l = {}^1/_2 C(t, x)\left[\Delta k_t^2 - (k_t - k_t^\circ)^2\right]. \qquad (2.6)$$

The system of equations obtained for B and C has the form

$$\frac{\partial B}{\partial t} + V_0 \frac{\partial B}{\partial x} = -\alpha_1 BC, \quad \frac{\partial C}{\partial t} = \beta_1 BC, \qquad (2.7)$$

where

$$\alpha_1 = \frac{e^2\omega_0^5}{8\pi m_e^2 k_t^{02}}, \quad \beta_1 = \frac{e^2\omega_0 k_t^\circ}{8\pi m_e^2}, \quad V_0 = V(k_t^\circ).$$

As in §1 this system may be solved by introducing the variable

$$\tau = \int\limits_0^t B(t', x)\, dt'.$$

We shall confine ourselves to presenting the final results

$$B(t, x) = B_0(\xi)\frac{s_1(x)\psi_1(\xi)}{1 - s_1(x) + s_1(x)\psi_1(\xi)},$$
$$C(t, x) = C_0(1 - s_1(x) + s_1(x)\psi_1(\xi))^{-1}, \qquad (2.8)$$

$$\xi = x - V_0 t; \quad s_1(x) = \exp(-2\alpha_1 C_0 x),$$
$$\psi_1(\xi) = \exp\left(\frac{\beta_1}{V_0}\int\limits_0^\xi B_0(\xi')\, d\xi'\right). \qquad (2.9)$$

In these formulas $B_0(\xi)$ determines the shape of the pulse incident on the plasma boundary, i.e.,

$$B(0, x) = B_0(x) \quad (x \leqslant 0,\ t = 0), \quad C(0, x) = C_0\theta(x).$$

Here C_0 is the initial level of the longitudinal noise.

3°. Finally we should consider the case when fairly long pulses pass through the plasma, for which the energy transfer of the plasma oscillations is appreciable. Leaving the general solution in this case we only note that for a long injection, for example N_0, if $t_0 > L/V_l$ (L is the width of the plasma layer, t_0 is the injection time), a stationary distribution of longitudinal waves is established. N^l increases from a minimum value at the plasma boundary to a maximum where saturation occurs, at a distance of V^l/γ, where γ is the time increment obtained in [2]. Here the maximum value of N^l is $V/V^l \approx 1/V_T^2$ times greater than for the time problem treated in paper [2].

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